PROJECT RISK MANAGEMENT
Paper 10

RISK ANALYSIS

PROBABILITY DISTRIBUTIONS

Random Variables

- 1. DISCRETE or CONTINUOUS random variables?
- a). shoe size of a randomly selected Australian man D
- b). foot size of a randomly selected Australian man. C
- c). time taken by a flight from New York to London C
- d). the birthday of Kevin Rudd- CONSTANT !

PROBABILITY DISTRIBUTIONS Definitions

- variable = different values in different situations
- random variable = value changes in no predictable manner.
- constant = retains same value from situation to situation

2 Types of random variables

Discrete Random Variable

- numerical values are limited to specific values within a range
- 'gaps' between values that variable can assume

Continuous Random Variable

- can take any value over a continuous range of values
- usually based on the *measure* of a quantity.

Continuous Random Variable

- In reality: data cannot be really continuous
- limitations to obtain accurate measurements
- data conceptually continuous, recorded data is discrete
- random variable of cost & time: commonly continuous
- e.g. cost element estimated to be between continuous \$100-\$150, although in 1^{C} steps

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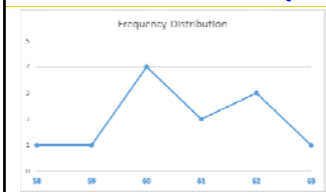
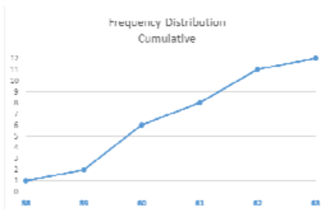
- 1. discrete or continuous random variables?
- a). shoe size of a randomly selected Australian man
- b). foot size of a randomly selected Australian man.
- c). Time taken by a flight from New York to London
- d). the birthday of Kevin Rudd

REMINDER – PAPER 8 – FREQUENCY DISTRIBUTION



Frequency Distribution

Cumulative Frequency Distribution



PROBABILITY DISTRIBUTIONS (PD)

- Probability Distribution for random variable:
  - Range of possible values (horizontal axis)
  - probability for these values (vertical axis)
- Total of probabilities in PD = 1
- Probabilities - structural, objective, subjective (Paper 9)
- Probability Distribution similar to Frequency Distribution
  - FD - outcomes that actually occurred in past
  - PD - outcomes that could occur in future
  - FD have mean & standard deviation.
  - PD have mean & standard deviation!!

PROBABILITY DISTRIBUTION - TYPES

DISCRETE PROBABILITY DISTRIBUTION

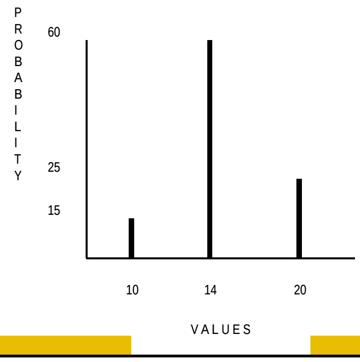
Expected Value (Mean)

- Measures of central tendency in probability distributions:
  - Mode, Mean (expected value) , Median
- Mode – peak of distribution, 'most likely', 'most probable'
- Expected value - mean value for distribution
- Median:
  - 50% percentile i.e. 50% probability of being exceeded or not being exceeded ie 50% of values above/below median
  - 50% of values to left, 50% to right, but not same total value

DISCRETE PROBABILITY DISTRIBUTION

⌘Every value assigned a probability

Value (days)	Probability %
10	15
14	60
20	25



Variance & Standard Deviation

- Variability of Probability Distribution is important
- Variability = Standard Deviation = √ variance
- measures - how much PD is spread from expected value (mean).

Discrete probability distribution
Expected Value (mean)

EV = weighted average of possible values

EV =Total of: (Each possible value X * by its Probability)

EV = Σ x. p(x)

x = possible value of random variable

P(x) = probability of x occurring

Calculate EV for this discrete probability distribution:

Value (days) x	Probability % P(x)
10	15
14	60
20	25

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EV - DISCRETE PROBABILITY DISTRIBUTION

Value (days)	Probability %
10	15
14	60
20	25

EV =

= 10(0.15) + 14 (0.60) + 20(0.25)

= 1.5 + 8.4 + 5.0

= 14.90 days

Discrete Random Variable - Standard Deviation

Value (days)	Probability
10	15
14	60
20	25

EV = 14.90 days

- Variance = Σ (x - EV)^2 * P(x) =
- = (10-14.9)^2 (.15) + (14-14.9)^2 (.60) + (20-14.9)^2 (.25)
  = 3.60 + 0.48 + 6.50
- V = 10.58
- Standard Deviation = $\sqrt{10.58}$ = 3.25 days

Discrete Random Variable - Standard Deviation

- SD indicates variability
- = average deviation from EV
- SD = $\sqrt{\text{variance}}$ = $\sqrt{\Sigma (x - EV)^2 * P(x)}$
- where
  - x value of the random variable
  - EV expected value of random variable
  - (x - EV) deviation from EV for each value
  - P(x) probability of x occurring

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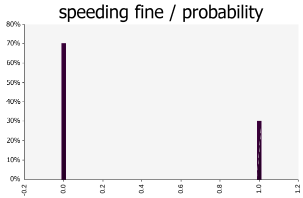
discrete probability distribution - *binomial*,

- yes/no distribution
- Only 2 values (yes/no) each which probability.
- EG: *probability* of risk event (eg speeding fine), either
  - no it will not (say 70% probability).
  - yes it will occur (say 30% probability)

speeding fine / probability



Discrete Random Variable - Standard Deviation

- What is variance & standard deviation for this project activity ?

Value (days)	Probability
10	15
14	60
20	25

- Variance = Σ (x- EV)^2 * P(x)
- Where:
  - x a value of the random variable
  - EV expected value of random variable.
  - (x- EV) deviation from EV for each value
  - P(x) probability of x occurring

How Australians use our time –(av hrs/day)

WOMEN		MEN	
Sleep	8.32	Sleep	8.30
Personal hygiene	0.56	Personal hygiene	0.45
Eating/Drinking	1.45	Eating/Drinking	1.38
Food preparation	1.08	Food preparation	0.29
Child care	2.39	Child care	1.06
Home maintenance	0.03	Home maintenance	0.14
Shopping	0.58	Shopping	0.38
Audio/Visual	4.08	Audio/Visual	4.34
Talking/writing	1.57	Talking/writing	1.29
Sport	0.18	Sport	0.26

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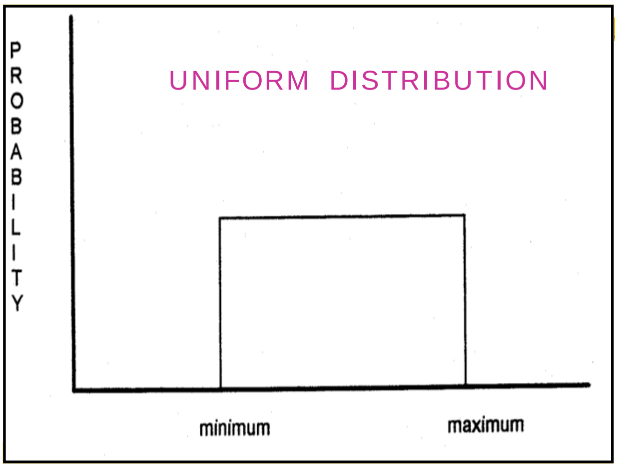
PROBABILITY DISTRIBUTION TYPES

CONTINUOUS PROBABILITY DISTRIBUTION



CONTINUOUS PROBABILITY DISTRIBUTION

- Continuous Random Variable = take any value over continuous range of values.
- Can't assign probability to one particular value!
- Continuous PD: probabilities by area
- Probabilities of all possible values = 1.
- So area under the curve =1.
- probability random variable will take a value between 2 values = *area under curve between these 2 values*.

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UNIFORM

- all values in a range within distribution have same probability
- Minimum & maximum values fixed
- all values between equally likely to occur
- useful "where you only have a min & max to go on"
- *very approximate model where very little or no available data*
- EV = average of 2 extremes
- no mode
- Variance = $\frac{(max - min)^2}{12}$
- in practice, limited application

CONTINUOUS PROBABILITY DISTRIBUTION

- difference between discrete & continuous random variables:
- Discrete:
  - probability can be assigned for each value
- Continuous:
  - probability interpreted by area
  - So, probability only considered in terms of between values
- Types – eg: uniform, triangular, normal, beta (PERT), etc

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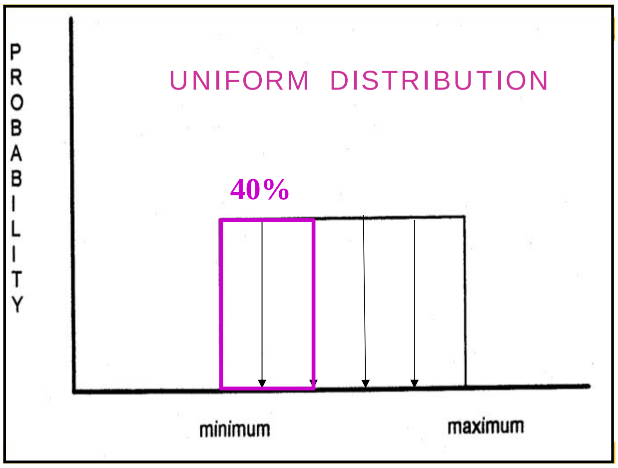
UNIFORM

- 1. The time to complete an activity varies from 5 to 10 minutes and every range of times has the same probability.
- This is shown in the uniform distribution below, where every 1-minute range has a probability of 0.2.
- *i. What is the probability that the time taken will be between 5 and 7 minutes?*
- *ii. What is the expected value ?*
- *iii. What is the variance?*



TRIANGULAR

- Used when minimum, maximum, most likely (mode) can be estimated
- Values near Min & Max less likely to occur than those near Most Likely
- most commonly used distribution for modelling expert opinion
- "sufficient in vast majority of practical situations"
- Good points
  - easy to specify minimum, maximum, most likely values;
  - does not require dealing with standard deviations
  - can be skewed. e.g. project durations have 1:2 skew.
  - eg: if ML=10, Min = 8 days, then max = 14 days.
- Problems
  - may cause significant underestimation of extreme values
  - where maximum difficult to determine, triangular not appropriate.
- Note: 'most likely' = mode/peak, although may assume = mean or median. Mean more appropriate but requires knowledge of all data within distribution, not practical, so mode used).

UNIFORM

- *i. What is the probability that the time taken will be between 5 and 7 minutes?*
- 40%
- *ii. What is the expected value ?*
- *(5+10)/2 = 7.5*
- ***Iii. What is the variance?***

$$\frac{(10-5)^2}{12} = \frac{25}{12} = 2.083$$

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TRIANGULAR

- 2. The total time that I wait for buses has a triangular distribution as shown below.
- *a). if I wait more than 15 minutes, I will be late for an appointment. What is the chance of this? (clue: probability measure by area)*
- *b). What is the mean (expected value) waiting time ?*
- *c) what is the variance? And standard deviation?*

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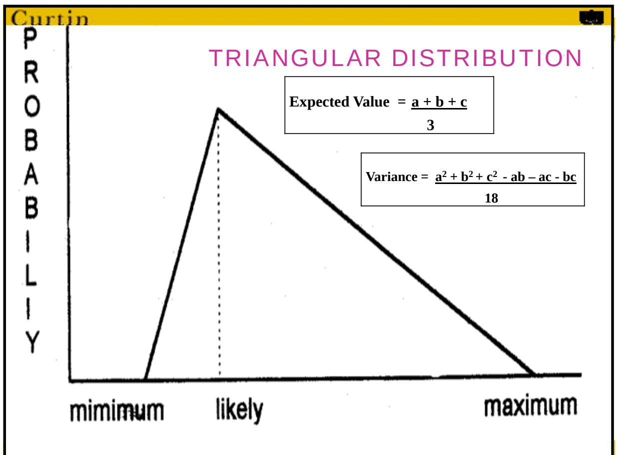

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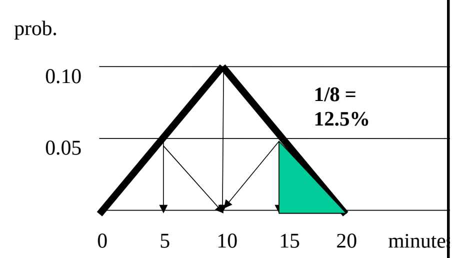


TRIANGULAR

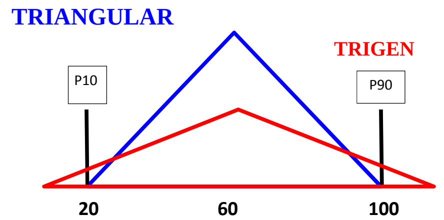
- *a). >15 minutes, late for appointment. What is chance of this?*
- *1/8, ie 12.5%*
- *b). mean waiting time ?*
- *= a+b+c /3 = 0 +10+20 /3 = 10 minutes*
- C) variance
- $a^2 + b^2 + c^2 - ab - ac - bc \;/ 18 =$
- $0^2 + 10^2 + 20^2 - (0)(10) - (0)(20) - (10)(20) \;/18 =$
- *300/18 = 16.66*
- *Standard Deviation = √ 16.66 = 4.08*

Trigen (triangular generation)

- people understate minimum & maximum values
- Min & Max in triangular = *absolute* extreme values,
- often difficult to estimate
- Trigen: Min & Max = 'practical' extremes
- decide probability they will be exceeded (eg P10-P90).

Trigen (triangular generation)

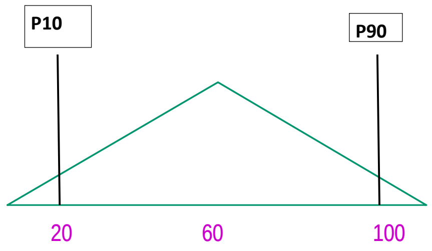
- variation of standard triangular distribution.
- three points — low, most likely, high.
- Lowest & highest - set at percentile levels, eg P10-P90
- EG Trigen (20,60,100; 10%,90%):
  - lowest value 20 (10% probability of being lower)
  - most likely 60
  - highest 100 (10% probability of being higher).



Triangular: Min/ML/Max: 20, 60, 100

- Trigen: decide Min and Max are at P10 & P90
- So, Values at P10 & P90 extended to values at P0 & P100,
- So widening overall range of distribution.

Trigen (triangular generation)

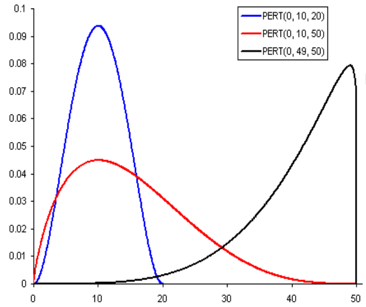


Trigen (20,60,100,10,90):

GUIDELINES FOR LIFE

- Never, under any circumstances, take a sleeping pill and a laxative on the same night.
- If you must choose between two evils, pick the one you've never tried before.
- Eat well; stay fit; die anyway.
- A balanced diet is a muffin in each hand.
- Junk is something you've kept for years and throw away three weeks before you need it.
- There is always one more idiot than you counted on.

BetaPERT Distribution (version of Beta)



Expected Value =

$$\frac{\text{Min} + 4\ \text{Most Likely} + \text{max}}{6}$$

Beta (PERT)

- EXPECTED VALUE
- Mean (EV) = (min + 4*most-likely + max)/6
- = (0 + 4*10 + 20)/6 = (60)/6 = 10
- *Triangular = Min+ML+Max /3 = 0 +10+20 /3 = 10*
- VARIANCE
- Variance = (EV-min)(max-EV) / 7
- = (10-0)(20-10)/7 =100/7 = 14.28. SD = √ 14.28 = 3.78
- *Triangular* variance = *16.66* *SD* = *4.08*

Beta PERT (Program Evaluation and Review Technique) Distribution

- Beta distributions - smooth curve, from zero likelihood, rounded peak, falling away to zero, can be skewed
- Beta requires obscure parameters to describe it.
- Special version of beta–BetaPERT: define by *min, most likely, max*
- Shape from 3 values – pragmatic, understandable, use expert opinion.
- Mean = (min + 4*most-likely + max)/6. Variance = (EV-min)(max-EV) / 7
- If skewed - less emphasis in direction of skew (compared to Triangular)
- EV: 4x more sensitive to ML, compared to triangular
- If variable has high max, relative to most likely & min, but has a low likelihood, then BetaPERT may be appropriate
- Triangular may be preferred - 'fat tail" (ie greater probability of pessimistic value), offsets people's unwillingness to state true pessimistic estimate. Conveys greater degree of uncertainty.
- Mean – 4x more sensitive to most likely value, compared to triangular where mean sensitive equally to all 3 values.

NORMAL

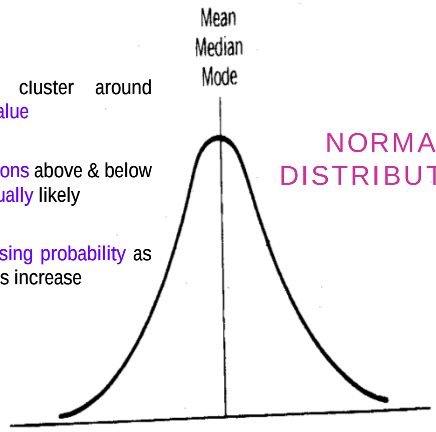
- "most important distribution in probability theory"
- variety of phenomena form distribution close to Normal
- reasonably good approximation for many situations.

Beta (PERT)

- The total time that I wait for buses has a PERT distribution:
- Min 0 minutes
- ML 10 minutes
- Max 20 minutes
- *a). What is the mean (expected value) waiting time ?*
- *c) what is the variance? And standard deviation?*

NORMAL DISTRIBUTION

- ⌘Values cluster around central value
- ⌘Deviations above & below being equally likely
- ⌘decreasing probability as deviations increase



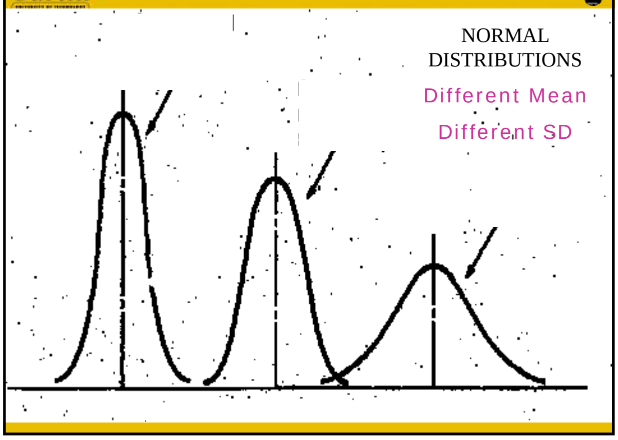
NORMAL

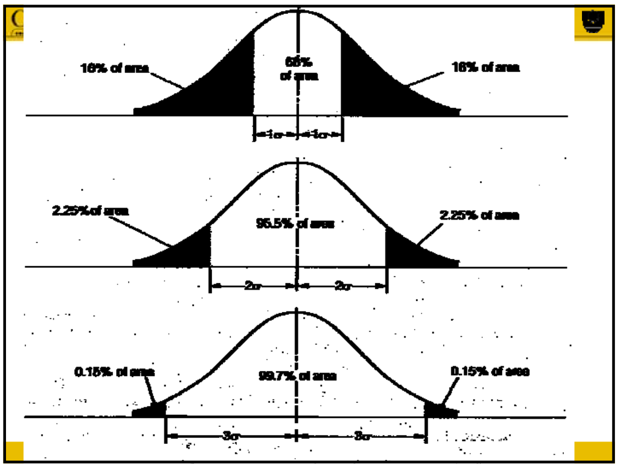
- 1 mean, median, mode - same value at centre of curve
- single peak, symmetrical around mean, bell-shaped,
- Shape of the curve determined by mean & standard deviation:
  - Mean determines height & location
  - standard deviation determines spread.
- probability that a variable will lie within :

  +/- 1 standard deviation of its mean is 68.3%

  +/- 2 standard deviations of its mean is 95.4%

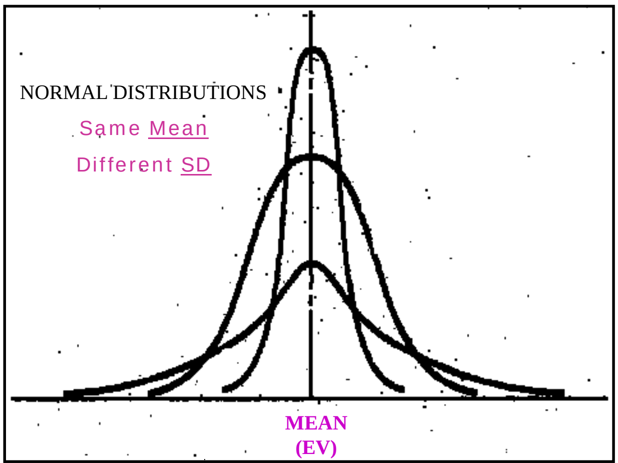
  +/- 3 standard deviations of its mean is 99.7%





NORMAL

- 3. A project activity is considered to be a normally distributed random variable with a:
  - mean of 120 hours
  - standard deviation of 3 hours.
- i. What is the probability of the project activity will be
  - a). *between 120 and 125 hours?*
  - B) > 125 hours



NORMAL DISTRIBUTION – Z SCORE

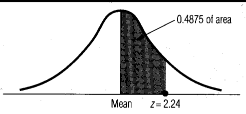
- probability project activity will not take more than 125 hours?
- 1. Locate value (125 hours) on ND (z score):

$$Z = \frac{x - \text{mean}}{SD}$$ where

x = value of interest on normal distribution curve

z = number of SDs from x to the mean

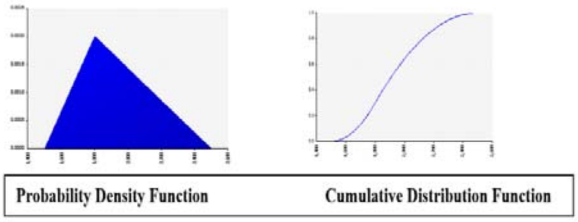
- $Z = \frac{(125-120)}{3}$
- $= 1.67$



*Areas Under ıne Standard Normal Probability Distribution Between the Mec and Positive Values of z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974

Cumulative Distribution Function



- *Probability density function* can be displayed *cumulative distribution function*
- CDF simply arithmetic summation of PDF.
- CDF answers question "what is probability that a continuous random variable will be less or equal to a stated value?"
- Many *CDFs* look like a stretched S shape
- difficult to identify its PDF form CDF shape

NORMAL DISTRIBUTION – Z SCORE

ii. Determine probability, based on area

Area between mean and z score of 1.67 = *0.4525* (Table)

- probability project activity will take 120-125 hours = 45.25%
- So probability it will take >125 hours = 50 – 45.25 = 4.75%

GOLDFISH TIME

1. Which probability distribution would you use, if

•You can predict a minimum and maximum possible values, but no most likely

•You can predict minimum, maximum and most likely

2. Why is the triangular distribution commonly used?

3.What is a continuous random variable?

4. What is the probability of a value X occurring for a continuous random variable

GUIDELINES FOR LIFE

- Experience is a wonderful thing. It enables you to recognise a mistake when you make it again.
- Thou shalt not weigh more than thy refrigerator.
- There is a very fine line between 'hobby' and 'mental illness.'
- You should not confuse your career with your life.
- Never lick a steak knife.